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I Semester M.Sc. Degree Examination, February 2019
(CBCS Scheme)
MATHEMATICS
Paper – M102T : Real-Analysis

Time : 3 Hours   Max. Marks : 70

Instructions : i) *Answer **any five** questions.*
ii) ***All** questions carry **equal** marks.*

1. a) Show that $f(x) = -x^2 \in R[0, C]$.

   b) Prove that $f \in R[\alpha]$ on [a, b] if and only if given $\varepsilon > 0$, $\exists$ a partition p of [a b] such that $U(p, f, \alpha) - L(p, f, \alpha) < \varepsilon$.

   c) If p^* is a refinement of partition p of [a, b] then show that $L(p, f, \alpha) \le L(p^*, f, \alpha) \le U(p^*, f, \alpha) \le U(p, f, \alpha)$. (4+5+5)

2. a) If $f \in R[\alpha]$ on [a, b] and $C \in R^+$, then prove that $Cf \in R[\alpha]$ on [a, b].

   b) If $f \in R[\alpha]$ on [a, b], $f \in [m, M]$ and ϕ is continuous function of f on [m, M] then prove that $\phi(t(x)) \in R[\alpha]$ on [a, b].

   c) If $f_1, f_2 \in R[\alpha]$ on [a, b] and $f_1 \le f_2$, then show that $\int_a^b f_1\, d\alpha \le \int_a^b f_2\, d\alpha$. (4+6+4)

3. a) Let f be Riemann integrable on [a, b] and let $F(x) = \int_a^x f(t)dt$, $a \le x \le b$ then prove that F is continuous on [a, b]. Further, show that f(t) is continuous at a point x_0 on [a, b]. Then F is differentiable at x_0 and $F'(x_0) = f(x_0)$.

   b) If $\lim_{\mu(p) \to 0} S(p, f, \alpha)$ exists, then show that $f \in R[\alpha]$ on [a, b] and $\lim_{\mu(p) \to 0} S(p, f, \alpha) = \int_a^b f d\alpha$.

   c) Calculate the total variation functions of $f(x) = x - [x]$ on [0, 2], where [x] is the maximum integral function. (6+4+4)

4. a) State and prove Weierstrauss M-test.

   b) Show that $\{e^{-nx}\}$ is uniformly convergent on [a, b].

   c) Suppose $f_n \to f$ uniformly on [a, b] and if $x_0 \in [a, b]$ such that $\lim_{x \to x_0} f_n(x) = A_n$ n = 1, 2, 3 ... then prove that

      i) An converges

      ii) $\lim_{x \to x_0} \lim_{n \to \infty} f_n(x) = \lim_{n \to \infty} \lim_{x \to x_0} f_n(x)$. (5+4+5)

P.T.O.

5. a) If $|f_n(x)| \leq M_n, \forall n \in N, \forall x \in [a, b]$ and $\sum_{n=1}^{\infty} M_n$ of positive reals is convergent, then prove that $\sum_{n=1}^{\infty} f_n(x)$ is uniformly convergent on [a, b].

b) Let $(f_n(x))$ be a sequence of functions uniformly convergent to f(x) on [a, b] and each $f_n(x) \in R[a, b]$. Then prove that $f(x) \in R[a, b]$, also prove $\lim_{n \to \infty} \int_a^x f_n(t)\, dt = \int_a^x f(t)\, dt \ \forall x \in [a, b]$.

c) Show that $\sum_{n=1}^{\infty} nxe^{-nx^2}$ converges point-wise and uniformly on [0, 4], k > 0. (5+5+4)

6. a) State and prove Stone-Weierstrauss theorem.

b) Define a k-cell in R^k prove that every k-cell is compact in R^k. (8+6)

7. a) Let $E \subset R^n$ be an open set and $f : E \to R^m$ is a map. Prove that if 'f' is continuously differentiable if and only if the partial derivatives $D_j f_i$ exists and are continuous on E for $1 \leq i \leq m, 1 \leq j \leq n$.

b) If $T_1, T_2 \in L(\mathbb{R}^n, \mathbb{R}^m)$, then prove that

i) $\| T_1 + T_2 \| \leq \| T_1 \| + \| T_2 \|$

ii) $\| \alpha T_1 \| = | \alpha | \| T_1 \|$.

c) Discuss the continuity of the function on $\mathbb{R}^2$ of

$$f(x, y) = \begin{cases} x^2 - y^2, & x \neq 0, \ y \neq 0 \\ 0, & x = 0, \ y = 0 \end{cases}$$ (7+4+3)

8. State and prove the implicit function theorem. 1

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