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I Semester M.Sc. Degree Examination, February 2019 (CBCS Scheme) MATHEMATICS Paper – M102T : Real-Analysis

Max. Marks: 70

Time: 3 Hours

Instructions: i) Answer any five questions.

ii) All questions carry equal marks.

a) Show that $f(x) = -x^2 \in R$ [0, C].

- Prove that $f \in R[\alpha]$ on [a, b] if and only if given $\epsilon > 0$, \exists a partition p of [a b] such that U $(p, f, \alpha) L(p, f, \alpha) < \epsilon$.
- c) If p* is a refinement of partition p of [a, b] then show that $L(p, f, \alpha) \le L(p^*, f, \alpha) \le U(p^*, f, \alpha) \le U(p, f, \alpha). \tag{4+5+5}$
- 2. a) If $f \in R[\alpha]$ on [a, b] and $C \in R^+$, then prove that $C f \in R[\alpha]$ on [a, b].
 - b) If $f \in R[\alpha]$ on [a, b], $f \in [m, M]$ and ϕ is continuous function of f on [m, M] then prove that $\phi(t(x)) \in R[\alpha]$ on [a, b].

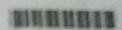
of If f_1 , $f_2 \in R[\alpha]$ on [a, b] and $f_1 \le f_2$, then show that $\int_a^b f_1 d\alpha \le \int_a^b f_2 d\alpha$. (4+6+4)

- 3. a) Let f be Riemann integrable on [a, b] and let $F(x) = \int f(t)dt$, $a \le x \le b$ then prove that F is continuous on [a, b]. Further, show that f(t) is continuous at a point x_0 on [a, b]. Then F is differentiable at x_0 and $F'(x_0) = f(x_0)$.
 - b) If $\lim_{\mu(p)\to 0} S(p,f,\alpha)$ exists, then show that $f\in R[\alpha]$ on [a,b] and $\lim_{\mu(p)\to 0} S(p,f,\alpha) = \int\limits_a^b fd\alpha$.
 - c) Calculate the total variation functions of f(x) = x [x] on [0, 2], where [x] is the maximum integral function. (6+4+4)
- 4: a) State and prove Weierstrauss M-test.
 - b) Show that {e-nx} is uniformly convergent on [a, b].
 - e) Suppose $f_n \to f$ uniformly on [a, b] and if $x_0 \in [a, b]$ such that $\lim_{x \to x_0} f_n(x) = A_n$ $n = 1, 2, 3 \dots$ then prove that
 - i) An converges

ii) $\lim_{x\to x_0} \lim_{n\to\infty} f_n(x) = \lim_{n\to\infty} \lim_{x\to x_0} f_n(x)$.

(5+4+5)

P.T.O.



- 5. a) If $|f_n(x)| \in M_n$, $\forall n \in N$, $\forall x \in [a,b]$ and $\sum_{n=1}^\infty M_n$ of positive reals is convergent, then prove that $\sum_{n=1}^\infty f_n(x)$ is uniformly convergent on [a,b].
 - b) Let $(f_n(x))$ be a sequence of functions uniformly convergent to f(x) on [a, b] and each $f_n(x) \in \mathbb{N}[a, b]$. Then prove that $f(x) \in \mathbb{N}[a, b]$, also prove $\lim_{n \to \infty} \int_{-\infty}^{\infty} f_n(t) \, dt = \int_{-\infty}^{\infty} f(t) \, dt \ \forall \ x \in [a, b]$.
 - g) Show that $\sum_{n=1}^{\infty} nxe^{-nx^2}$ converges point-wise and uniformly on [0, 4], k > 0. (5+5+4)
- 6. a) State and prove Stone-Weierstrauss theorem.
 - b) Define a k-cell in Rk prove that every k-cell is compact in Rk.
- (8÷6)
- 7. (a) Let E ⊂ Rⁿ be an open set and f : E → R^m is a map. Prove that if 'f is continuously differentiable if and only if the partial derivatives Df exists and are continuous on E for 1 ≤ i ≤ m, 1 ≤ j ≤ n.
 - p) If T_1 , $T_2 \in L(\mathbb{R}^n, \mathbb{R}^m)$, then prove that

$$|T_1 + T_2| \le |T_1| + |T_2|$$

|| ||
$$|| \alpha T_1 || = || \alpha || T_1 ||$$
.

g) Discuss the continuity of the function on IR2 of

$$f(x, y) = \begin{cases} x^2 - y^2, & x \neq 0, & y \neq 0 \\ 0, & x = 0, & y = 0 \end{cases}$$

8. State and prove the implicit function theorem.