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First Semester M.Sc. Examination, February 2019
(CBCS Scheme)
MATHEMATICS
M104T : Ordinary Differential Equations

Time : 3 Hours

Max. Marks : 70

*Instructions : 1) All questions have **equal** marks.*
*2) Answer **any five** questions.*

1. a) Let $y_1, y_2, y_3, \ldots, y_n$ be the n-solutions of $L_n\, y = 0$ on I with initial conditions $y_j^{(i-1)}(x_0) = a_{ij}$, $1 \le i, j \le n$, where all a_{ij} are some constants and $x_0 \in I$. Then show that the necessary and sufficient conditions for $y_1, y_2, y_3, \ldots, y_n$ to form fundamental set is $|a_{ij}| \neq 0$.

   b) Find the Wronskian of $y''''' - y'''' - y' + y = 0$. (7+7)

2. a) Derive an expression that converts adjoint differential equation into a standard differential equation. For a standard differential equation which is not self-adjoint, obtain a differential equation which is self-adjoint.

   b) Prove that the operator $L. = \frac{d^k}{dx^k} P(x) \frac{d^k}{dx^k}$ is self-adjoint. (7+7)

3. a) Show that $y'' + \left(\frac{1}{4x^2} + \frac{k}{(x \log x)^2}\right) y = 0$, $k > 0$ is a constant, is oscillatory if $k > \frac{1}{4}$ and non-oscillatory if $k < \frac{1}{4}$.

   b) Show that a solution of $y'(x) = \begin{cases} \frac{4x^3 y}{x^4 + y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$ is not unique. (9+5)

4. a) Define the Sturm-Liouville eigenvalue problem. Also find all the eigenvalues and eigenfunctions of $\frac{d}{dx}\left(\frac{1}{3x^2+1} \cdot y^1\right) + \lambda\,(3x^2+1)y = 0$ with $y(0) = 0 = y(\pi)$.

b) Establish the Green's function for the solution of eigenvalue problem. (7+7)

5. a) Define different types of singular points of $p(x)y'' + q(x)y' + r(x)y = 0$. Also obtain the corresponding differential equation for the point at infinity.

b) Show that Hermite polynomials are orthogonal over $(-\infty, \infty)$. (7+7)

6. a) Obtain the general solution of Chebyshev differential equation.

b) Obtain the general solution of Gauss-Hypergeometric differential equation about $x = 0$. (7+7)

7. Define the fundamental matrix of $\underset{\sim}{X}' = \underset{\sim}{A}\,\underset{\sim}{X}$. Hence obtain the general solution when $\underset{\sim}{A} = \begin{bmatrix} 7 & -1 & 6 \\ -10 & 4 & -12 \\ -2 & 1 & -1 \end{bmatrix}, \underset{\sim}{X} = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}$. 14

8. a) Determine the nature and stability of the critical point of $\frac{dx}{dt} = 2x - 2y + 11; \frac{dy}{dt} = 11x - 8y + 49$.

b) Determine the stability of the critical point (0, 0) of the following system.

$\frac{dx}{dt} = -x + y^2; \frac{dy}{dt} = -y + x^2$ by constructing the Liapunove function. (7+7)