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60854

First Semester M.Sc. Examination, February 2019  
(CBCS Scheme)

MATHEMATICS

M104T : Ordinary Differential Equations

Time : 3 Hours

Max. Marks : 70

**Instructions :** 1) All questions have equal marks.

2) Answer any five questions.

1. a) Let  $y_1, y_2, y_3, \dots, y_n$  be the  $n$ -solutions of  $L_n y = 0$  on  $I$  with initial conditions  $y_j^{(i-1)}(x_0) = a_{ij}$ ,  $1 \leq i, j \leq n$ , where all  $a_{ij}$  are some constants and  $x_0 \in I$ . Then show that the necessary and sufficient conditions for  $y_1, y_2, y_3, \dots, y_n$  to form fundamental set is  $|a_{ij}| \neq 0$ .

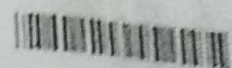
b) Find the Wronskian of  $y'''' - y'''' - y' + y = 0$ . (7+7)

2. a) Derive an expression that converts adjoint differential equation into a standard differential equation. For a standard differential equation which is not self-adjoint, obtain a differential equation which is self-adjoint.

b) Prove that the operator  $L = \frac{d^k}{dx^k} P(x) \frac{d^k}{dx^k}$  is self-adjoint. (7+7)

3. a) Show that  $y'' + \left( \frac{1}{4x^2} + \frac{k}{(x \log x)^2} \right) y = 0$ ,  $k > 0$  is a constant, is oscillatory if  $k > \frac{1}{4}$  and non-oscillatory if  $k < \frac{1}{4}$ .

b) Show that a solution of  $y'(x) = \begin{cases} \frac{4x^3 y}{x^4 + y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$  is not unique. (9+5)



4. a) Define the Sturm-Liouville eigenvalue problem. Also find all the eigenvalues and eigenfunctions of  $\frac{d}{dx} \left( \frac{1}{3x^2 + 1} \cdot y' \right) + \lambda (3x^2 + 1)y = 0$  with  $y(0) = 0 = y(\pi)$ .
- b) Establish the Green's function for the solution of eigenvalue problem. (7+7)
5. a) Define different types of singular points of  $p(x)y'' + q(x)y' + r(x)y = 0$ . Also obtain the corresponding differential equation for the point at infinity.
- b) Show that Hermite polynomials are orthogonal over  $(-\infty, \infty)$ . (7+7)
6. a) Obtain the general solution of Chebyshev differential equation.
- b) Obtain the general solution of Gauss-Hypergeometric differential equation about  $x = 0$ . (7+7)
7. Define the fundamental matrix of  $\underline{X}' = \underline{A} \underline{X}$ . Hence obtain the general solution

$$\text{when } \underline{A} = \begin{bmatrix} 7 & -1 & 6 \\ -10 & 4 & -12 \\ -2 & 1 & -1 \end{bmatrix}, \underline{X} = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}.$$

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8. a) Determine the nature and stability of the critical point of

$$\frac{dx}{dt} = 2x - 2y + 11; \frac{dy}{dt} = 11x - 8y + 49.$$

- b) Determine the stability of the critical point  $(0, 0)$  of the following system.

$$\frac{dx}{dt} = -x + y^2; \frac{dy}{dt} = -y + x^2 \text{ by constructing the Liapunov function. (7+7)}$$