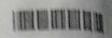
## I Semester M.Sc. Examination, February 2019 (CBCS Scheme) MATHEMATICS M107SC: Mathematical Analysis

Time: 3 Hours

Max. Marks: 70

Instructions: 1) Answer any five questions.

- 2) All questions have equal marks.
- a) Let (X, d<sub>x</sub>) and (Y, d<sub>y</sub>) be metric spaces, E ⊂ X and p ∈ E. The function f is continuous at p, iff for every sequence {x<sub>n</sub>} in E converging to P, then show that the sequence {f (x<sub>n</sub>)} in Y converges to f (p).
  - b) Show that continuous function on a compact metric space is uniformly continuous. (7+7)
- 2-Define connected metric space. Prove that a continuous mapping from a connected metric space into a metric space is connected. (14)
- 3. a) If f(x) is continuous on [a, b], f'(c) exists at some point c ∈ [a, b], g(x) is defined on an interval I which contains the range of f(x) and g (x) is differentiable at the point f(c), if h(x) = g(f(x)), then show that h(x) is differentiable at c.
  - b) With help of function  $f(x) = x^2 \sin(1/x)$ , prove the continuity of derivatives. (7+7)
- 4. a) State and prove L' Hospitals rule for indeterminate form of type  $\infty/\infty$ .



- b) If f (x) is real valued continuous function on [a, b] which is differentiable in (a, b) then show that there exist a point  $x \in (a, b)$  such that f(b) f(a) = (b a) f'(x). (7+7)
- a) Every bounded sequence of real number contains a convergent subsequence.
   Prove or disprove it.
  - b) Prove that a sequence of real number converges iff it is Cauchy sequence. (7+7)
- 6. a) Show that e is irrational.

(7+7) Prove: 
$$\lim_{n \to \infty} \left( 1 + \frac{1}{n} \right)^n = e$$
.

- 7. a) State and prove Cauchy integral test.
  - b) Given a series  $\sum_{n=1}^{\infty} a_n$

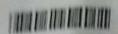
Let 
$$\alpha = \lim_{n \to \infty} \sup \sqrt[1]{|a_n|}$$

then prove that

- i)  $\sum_{n=1}^{\infty} a_n$  converges if  $\alpha < 1$
- ii)  $\sum_{n=1}^{\infty} a_n$  diverges if  $\alpha > 1$
- iii) if  $\alpha = 1$ , the test gives no information.
- c) Test the convergence of

$$\sum_{n=1}^{\infty} \left( \sqrt{n+1} - \sqrt{n-1} \right)$$

(5+5+4)



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- g. a) State and prove Martin's theorem.
  - b) Suppose
    - i)  $\sum_{n=0}^{\infty} a_n$  converges absolutely
    - ii)  $\sum_{n=0}^{\infty} a_n = A$
    - iii)  $\sum_{n=0}^{\infty} b_n = B$
  - iv)  $C_n = \sum_{k=0}^n a_k b_{n-k}$ , n = 0, 1, 2 ...

then show that

$$\sum_{n=0}^{\infty} C_n = AB$$

c) Assume that each  $a_n \ge 0$ , n = 1, 2, ..., then the product  $\prod_{n=1}^{\infty} (1 + a_n)$  converges. (5+5+4)