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I Semester M.Sc. Degree Examination, February 2019
(CBCS Scheme)
MATHEMATICS
Paper : M105T :Discrete Mathematics

Time : 3 Hours

Max. Marks : 70

***Instructions** : i) Answer **any five full** questions.*
*ii) **All** questions carry **equal marks.***

1. a) Explain methods of proof with examples. Prove or disprove the following statement "There is no greatest prime integer" using any of the proof techniques.

   b) Test the validity of the statements : "Either Vikram will run or Anchal will speak.
   If Anchal speaks, then Harish will fly and the rose is purple.
   The rose is not purple
   Thus, Vikram will run".

   c) Obtain the principal conjunctive normal form of the proposition $(\sim p \to r) \wedge (q \leftrightarrow p)$ without writing the truth table. Further write its negation. **(4+5+5)**

2. a) Show that there are at least 90 ways to choose six numbers from 1 to 5 so that the choices have the same sum.

   b) A big bag contains many red marbles, many white marbles and many blue marbles. What is the least number of marbles one should take out to be sure of getting at least six marbles of the same color ?

   c) How many different positive integers can be obtained as a sum of two or more numbers 1, 3, 5, 10, 20, 50, 82 ?

(5+5+4)

P.T.O.

3. a) Model the rabit population problem using recurrence relations and solve it explicitly.

b) How many ways are there to distribute 25 identical balls into seven distinct boxes if the first box can have no more than 10 balls but any amount can go in each of the other six boxes ?

c) If a leading digit 0 is permitted, find the number of 25-digit binary sequences that can be formed using an even number of zeros and odd number of ones.

(4+5+5)

4. a) Given a set A, with $|A| = n$, and R be a relation on A. Let M be the matrix of R then prove that

i) R is reflexive if and only if $I_n \leq M$, where I_n is the identity matrix of order n.

ii) R is transitive if and only if $M^T \leq M$, where M^T is the transpose of M.

iii) R is anti-symmetric if and only if $M \cap M^T \leq I_n$.

b) Let R be a relation on a set A = {a, b, c, d, e}, defined as R = {(a, a), (b, c), (a, b), (a, d), (a, e), (c, b), (b, d), (d, e), (e, d)}. Find the transitive closure of R using Warshall's algorithm.

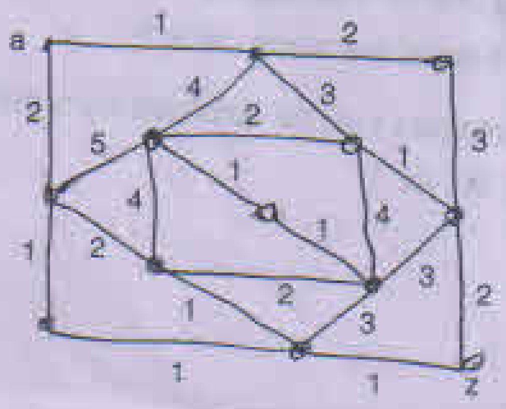
c) Define a boolean algebra. Show that in a boolean algebra for any two elements x and y, x = y if and only if $(x \wedge \overline{y}) \vee (\overline{x} \wedge y) = 0$.

(4+5+5)

5. a) Define complement of a graph. Prove that two graphs are isomorphic if and only if their complements are isomorphic.

b) Define a component of a graph. Prove that a simple graph with p vertices and K components has size atmost $\frac{(p-k)(p-k+1)}{2}$.

c) Applying Dijkstra's algorithm, find a shortest-path between 'a' and 'z' for the following weighted graph



(4+5+5)

6. a) Define an Eulerian graph with an example. Prove that a graph is Eulerian if and only if it can be decomposed into edge disjoint cycles.

   b) What do you mean by Hamiltonicity in graphs ? Show that any k-regular simple graph with 2k-1 vertices is Hamiltonian.

   c) Explain the nearest neighbor method using an example and hence find the weight of a spanning cycle for the considered example using the method.

(4+5+5)

7. a) Define a planar graph. Show that K_5 and $K_{3,3}$ are non-planar.

   b) If G is a triangle-free planar graph then prove that its size is atmost 2p-4, where p is the order of the graph G.

   c) Define each term of the following and establish $\alpha_1(G) + \beta_1(G) = p$. (5+4+5)

8. a) Define a tree. Prove that a graph with p vertices is a tree if and only if it has $(p-1)$ edges.

   b) Define a m-array tree. Prove that a full m-array tree with i branch vertices has mi + 1 vertices.

   c) By applying Prim's algorithm obtain a minimum spanning tree for the following graph :

(4+5+5)