I Semester M.Sc. Degree Examination, February 2019 (CBCS Scheme) MATHEMATICS Paper - M 101 T: Algebra - I

Max. Marks: 70

Time: 3 Hours

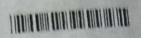
Instructions: i) Answer any five questions.

ii) All questions carry equal marks.

- 1. a) Show that $S_n/A_n = \{1, -1\}$ where S_n is a symmetric group and A_n is
 - b) If N and M are normal subgroup of a group G, prove that NM/M ≈ N/N ∩ M. (4+5+5)
 - c) State and prove the Cayley's theorem for permutation groups.
- 2. a) Let G be a finite group and S is finite G-set. If $x \in S$ then show that $O(G_x) = O(G)/O(stab(x)).$
 - b) Derive the class equation for finite groups.
 - c) By using generator-relation form of S3, verify the class equation of S3, where (5+4+5)S₃ is a symmetric group.
- 3. a) State and prove the Cauchy's theorem for abelian group.
 - b) Show that the number of p-sylow subgroups of G for a given prime is congruent to 1 modulo p.
 - c) Show that a group of order pq with p and q are distinct primes such that (5+5+4)p < q and $q \neq (mod p)$ is abelian.
- 4 a) Define a solvable group. Prove that every subgroup of a solvable group is solvable.
 - b) State and prove Jordan-Holder theorem.

5. a) Let R be a commutative ring with unity whose only ideals are {0} and R itself. Then prove that R is field.

- b) Show that the homomorphism φ of R onto R¹ is an isomorphic if and only if $Ker \phi = \{0\}.$
- c) Let R and R¹ be rings and φ is a homomorphism of R and R¹ with kernel U. Then show that $R^1 \approx R/U$.



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- 6. a) Define a principal ideal ring. Show that a ring Z of integer is a principal ideal
 - Define maximal ideal of a ring. If R is commutative ring with element and M is an ideal of R, then show that M is a maximal ideal of R if and only if
 - Show that any two isomorphic integral domain have isomorphic quotient (5+5+4)fields.
- 7. a) Let R be an Euclidean ring. Show that any ideal $A = (a_0)$ is maximal ideal in R if and only if a is a prime element of R.
 - b) Show that every Euclidean ring is a principal ideal ring.
 - State and prove the unique factorization theorem.

(5+4+5)

- % a) Prove that deg (fg) = deg (f) + deg (g) for f, g ∈ R [x]. Further if R is an integral domain, then show that R [x] is also an integral domain.
 - b) State and prove the Gauss lemma.
 - c) If p is a prime number, prove that the polynomial $x^n p$ is irreducible over Q.

(5+5+4)