PG – 253

I Semester M.Sc. Degree Examination, January/February 2018
(CBCS Scheme)
MATHEMATICS
M104T : Ordinary Differential Equations

Time : 3 Hours Max. Marks : 70

Instruction : *Answer* ***any five*** *questions.* ***All*** *questions carry* ***equal*** *marks.*

1. a) Show that $\{\psi_i(x), i = 1 \text{ to } n\}$ forms a fundamental set for $L_n y = 0$ on I if and only if $W\{\psi_i(x), i = 1 \text{ to } n\} \neq 0, \forall x \in I$. 9

   b) If the Wronskian of $\phi_1(x)$ and $\phi_2(x)$ is $3e^{4x}$ and if $\phi_1(x) = e^{2x}$, then find $\phi_2(x)$. 5

2. a) Explain the method of variation of parameters of solving $L_n y = b(x)$. 9

   b) Verify Lagrange's identity for $x^2 y'' + 9xy' + 12y = 0$. 5

3. a) State the existence and uniqueness theorem on the solution of $y' = f(x, y) : y(x_0) = y_0$ and hence illustrate it for $y' = y^2, y(1) = -1$ in the domain $|x - 1| \leq a, |y + 1| \leq b$, where a and b are constants. 7

   b) State and prove Sturm's separation theorem on the zeros of the solutions of a self-adjoint differential equation. 7

4. a) Establish an eigen function expansion formula and hence expand sin x in terms of orthonormal eigen functions of $y'' + \lambda y = 0 : y(0) = 0 = y(\pi)\ (\lambda > 0)$ 9

   b) Construct Green's function for $y'' + \frac{1}{4}y = \sin 2x\ ;\ y(0) = 0 = y(\pi)$. 5

5. a) Find the ordinary, regular and irregular singular points (including ∞) of $x(1 - x)y'' + \{c - (a + b + 1)x\}y' - aby = 0$, where a, b and c are constants. 7

   b) Find the Frobenius series solution of $(2x + x^3)y'' - y' - 6xy = 0$ about $x = 0$. 7

6. a) Obtain the general solution of Hermite's differential equation. 6

   b) Prove the following :

      i) $(n + 1) L_{n+1}(x) = (2n + 1 - x) L_n(x) - nL_{n-1}(x)\ (n \geq 1)$.

      ii) $T_{n+1}(x) - 2xT_n(x) + T_{n-1}(x) = 0\ (n \geq 1)$. 8

P.T.O.

7. Find the fundamental matrix and the general solution of

$$\frac{d\overline{X}}{dt} = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 3 & -4 \\ 4 & 1 & -4 \end{bmatrix} \overline{X},\ \overline{X} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}.$$

Also, show that the solutions are linearly independent. 14

8. a) Explain different types of critical points of

$$\frac{dx}{dt} = ax + by\ ;\ \frac{dy}{dt} = cx + dy\ (ad - bc \neq 0).$$ 8

b) Find the stability of the critical point (0, 0) of the system.

$$\frac{dx}{dt} = -x + y^2$$

$$\frac{dy}{dt} = -y + x^2$$

by constructing the Liapunov function. 6

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