PG – 254

I Semester M.Sc. Degree Examination, Jan./Feb. 2018
(CBCS Scheme)
MATHEMATICS
M105T : Discrete Mathematics

Time : 3 Hours Max. Marks : 70

Instructions : *i) Answer **any five full** questions.*
*ii) **All** questions carry **equal** marks.*

1. a) Explain methods of proof with examples.

b) Test the validity of the following arguments.

"If there was a cricket match, then travelling was difficult. If they arrived on time, then travelling was not difficult. They arrived on time. Hence, there was no cricket match".

c) If $S = (\sim p \vee \sim q) \rightarrow (p \leftrightarrow \sim q)$, then find its principal disjunctive normal form.

(4+5+5)

2. a) Suppose a patient is given a prescription of 45 pills with instruction to take at least one pill a day for 30 days. Prove that there must exist a period of consecutive days which the patient takes a total of 14 pills.

b) How many ways are there to distribute three different teddy bears and nine identical lollipops to four children ?

i) Without restriction.

ii) With no child getting two or more teddy bears.

c) How many arrangements are there of TINKERER with two but not three consecutive vowels ? **(5+5+4)**

3. a) Solve the recurrence relation of the tower of Hanoi problem.

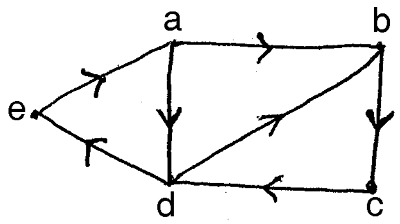
b) Solve the recurrence relation $a_n - 5a_{n-1} + 6a_{n-2} + 9a_{n-3} = 3n^2 + 2$ with $a_0 = 2$, $a_1 = -2$, $a_2 = 4$.

c) Using generating functions solve $a_{n+1} - a_n = 3^n$, $a_0 = 1$. **(4+5+5)**

P.T.O.

4. a) Define connectivity and reachability relations. Prove that $R^{\infty} = R \cup R^2 \cup \ldots \cup R^n$, for a relation R defined on a set A such that $|A| = n$.

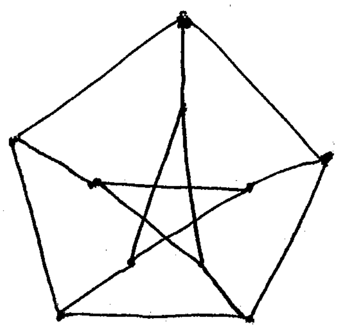
b) Define transitive closure of a relation and find the transitive closure of the relation :



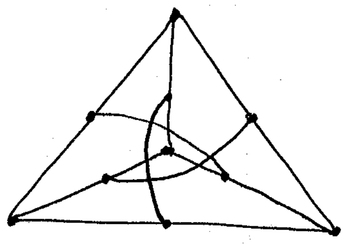
c) Define POSET. If A = {1, 2, 3, 4, 6, 8, 9, 12, 18, 24} be ordered by divisibility then draw the Hasse diagram of the POSET. **(4+5+5)**

5. a) State and prove first theorem in graph theory, further, show that in any graph G, the number of vertices in odd degree is even.

b) Define isomorphism graphs. Verify the following graphs are isometric or not.



G_1



G_2

c) Define self complementary graphs. Prove that any self-complementary graph has 4n or 4n + 1 vertices for $n \geq 1$. **(4+5+5)**

6. a) Let G be a connected graph. Then show that G contains an Eulerian trial if and only if G has exactly 2 odd vertices.

b) State and prove Dirac theorem for Hamiltonian graph.

c) Write a short note on the following :

i) Konigs Berg bridge problem

ii) Travelling salesman problem

iii) Nearest neighbour method. **(4+5+5)**

7. a) Show that any connected plane (p, q) - graph (p ≥ 3) with r faces $q \geq \frac{3r}{2}$ and $q \leq 3p - 6$.

b) Define vertex and edge connectivity of a graph. Prove the following identity with usual notations $K(G) \leq \lambda(G) \leq \delta(G)$.

c) Define the following :

i) Covering number of a graph $\alpha_0(G)$.

ii) Independence number of a graph $\beta_0(G)$. Find $\alpha_0(K_p), \beta_0(K_p), \alpha_0(C_p)$ and $\beta_0(C_p)$. **(5+5+4)**

8. a) Show that every non-trivial (p, q) – tree T contains atleast two vertices of degree 1.

b) Define binary tree with an example. Prove the following binary tree with p ≥ 3 vertices.

i) The number of vertices is always odd.

ii) The number of pendent vertices is $\frac{p+1}{2}$.

c) Define minimal spanning tree. Explain Krushkal's algorithm with an example. **(3+6+5)**