PG – 855

I Semester M.Sc. Examination, January 2017
(R.N.S.) (2011 Onwards)
MATHEMATICS
M – 102 : Real Analysis

Time : 3 Hours Max. Marks : 80

Instructions : *1) Answer* ***any five*** *questions, choosing atleast* ***one*** *from* ***each*** *Part.*

2) ***All*** *questions carry* ***equal*** *marks.*

PART – A

1\. a) Show that $f(x) = -x^2 \in R[0, c]$. **4**

b) If $f \in R[\alpha]$ on [a, b], then prove that $\underline{\int_a^b} f\,d\alpha = \overline{\int_a^b} f\,d\alpha = \int_a^b f\,d\alpha = \lambda[\alpha(b) - \alpha(a)]$,

where $\lambda \in [m, M]$ (m : greatest lower bound and M : least upper bound). **6**

c) Prove that $f \in R[\alpha]$ on [a, b] iff

given $\in > 0$, $\exists$ a partition P of [a, b]/$U(P, f, \alpha) - L(P, f, \alpha) < \in$. **6**

2\. a) If $f_1, f_2 \in R[\alpha]$ on [a, b] and $f_1 \leq f_2$, then show that $\int_a^b f_1\,d\alpha \leq \int_a^b f_2\,d\alpha$. **4**

b) If $f \in R[\alpha]$ on [a, b], $f \in [m, M]$ and ϕ is a continuous function of f on [m, M], then show that $\phi(f(x)) \in R[\alpha]$ on [a, b]. **8**

c) Show that $|f| \in R[\alpha]$ on [a, b] if $f \in R[\alpha]$ on [a, b]. Give an example of a function f such that $|f| \in R[\alpha]$ on [0, 1] and $f \notin R[\alpha]$ on [0, 1]. **4**

3\. a) Let $f \in R[a, b]$ and let $F(x) = \int_a^x f(t)\,dt\ [a \leq x \leq b]$. Then prove that F(x) is continuous on [a, b]. Further, show that if f(x) is continuous at x_0 in [a, b], then F is differentiable and $F'(x_0) = f(x_0)$. **8**

b) If $\lim\limits_{\mu(p)\to 0} S(P, f, \alpha)$ exists, then show that $f \in R[\alpha]$ on [a, b] and

$\lim\limits_{\mu(p)\to 0} S(P, f, \alpha) = \int_a^b f d\alpha$. **4**

c) Show that a function of bounded variation on [a, b] is bounded. **4**

PART – B

4. State and prove Cauchy's principle for uniform convergence of

a) $\{f_n(x)\}$ on [a, b],

b) $\sum\limits_{n=1}^{\infty} f_n(x)$ on [a, b]. **16**

5. a) Let $\{f_n(x)\}$ be uniformly convergent to f(x) on [a, b] and let each $f_n(x)$ be continuous on [a, b]. Prove that f(x) is continuous on [a, b]. **8**

b) Discuss the properties of any two of exponential, logarithmic and Fourier series. **8**

6. State and prove Stone-Weierstrass theorem. **16**

PART – C

7. a) Let E be an open subset of R^n and $f : E \to R^n$ be a differentiable function at $x_0 \in E$. Then prove that f is continuous at x_0 and $f'(x_0)$ is unique. **6**

b) If $T \in L(\mathbb{R}^n, \mathbb{R}^m)$, then prove that $||T|| < \infty$ and T is a uniformly continuous mapping of $\mathbb{R}^n$ onto $\mathbb{R}^m$. **6**

c) Discuss the continuity on R^2 of $f(x, y) = \begin{cases} x^2 - y^2 & , x \neq 0, \ y \neq 0 \\ 0 & , x = 0, \ y = 0 \end{cases}$. **4**

8. State and prove the implicit function theorem. **16**

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