I Semester M.Sc. Degree Examination, January 2017
(R.N.S.) (2011 Onwards)
MATHEMATICS
M104 : Ordinary Differential Equations

Time : 3 Hours Max. Marks : 80

Instructions : 1) ***All*** *questions carry* ***equal*** *marks.*
2) *Solve* ***any five*** *questions choosing atleast* ***two*** *from* ***each*** *Part.*

PART – A

1. a) Prove that the n solutions $\{\phi_j \cdot (x), j = 1 \text{ to } n\}$ of $L_n y = 0$ on I are linearly independent iff $W\{\phi_j \cdot (x), j = 1 \text{ to } n\} \neq 0$ for all $x \in I$. **7**

b) Using Liouville's formula, compute the Wronskian of the independent solutions of $y^{(5)} - y^{(4)} - y^1 + y = 0$ in [0, 1]. **6**

c) Define adjoint and self-adjoint differential equations. **3**

2. a) Establish the Green's formula for L_n the operator L_n and hence prove that $L_n^{**} = L_n$, where L^*_n is the adjoint operator of L_n. **7**

b) State and prove Sturm's separation theorem on the zeros of the solutions of a self-adjoint differential equation. Show that zeros of sin(logx) and cos(logx) separate each other. **9**

3. a) Explain the method of variation of parameters of finding the general solution of $L_n y = b(x)$. **9**

b) State the existence and uniqueness theorem on the solution of the IVP : $y' = f(x, y);\ y(x_0) = y_0$ on the interval $|x - x_0| \leq h$.

Find the value of 'h' for the IVP : $y' = y^2$, $y(1) = -1$. **7**

P.T.O.

4. a) Define an eigenvalue problem. Find the eigenvalues and eigen functions of $\frac{d}{dx}(xy') + \frac{\lambda}{x} y = 0;\ y'(1) = 0 = y'(e^{2\pi})$. Where λ is non-negative. Expand logx in terms of orthonormal eigen functions of the above eigenvalue problem. **10**

b) Obtain the Green's function for the eigenvalue problem $y'' + \lambda y = b(x); y(0) = 0 = y(\pi)$. **6**

PART – B

5. a) Find the series solution of the Hermite differential equation, $y'' - 2xy' + 2ny = 0$, about the ordinary point x = 0, where n is non-negative integer. Find its general solution. **6**

b) Using the Frobenius method, obtain the general solution of the equation $x^2y'' - xy' + (x^2 - 3)\,y = 0$, about the regular singular point x = 0. **6**

c) Prove that, $\frac{1}{1-t} \exp\left\{-\frac{tx}{1-t}\right\} = \sum_{n=0}^{\infty} L_n(x) t^n$. **4**

6. a) Find all the singular points of the equation,

$x(1-x)y'' + [c - (a+b+1)x]y' - aby = 0$. Find the indicial equations and the exponents about each of the regular singular points. Put $x = \frac{1}{t}$ and transform the equation and determine the type of singular point at infinity. **7**

b) Determine the Chebyshev polynomial solution of $(1-x^2)y'' - xy' + n^2y = 0$. **6**

c) Prove that $(1-x^2)\,T_n^{1}(x) + nxT_n(x) - nT_{n-1}(x) = 0$. **3**

7. a) Find the fundamental matrix solution of $\frac{dx}{dt} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 3 & 0 \\ 0 & 1 & 1 \end{bmatrix} x$. **5**

b) Determine the nature and stability of the critical point in the following :

i) $\frac{dx}{dt} = -2x + 4y, \ \frac{dy}{dt} = -2x + 6y$

ii) $\frac{d^2x}{dt^2} = +2b\frac{dx}{dt} + a^2x = 0$ when b > a are real constants. **(3+3)**

c) Determine the nature and stability of the critical point (0, 0) in the following :

$\frac{dx}{dt} = x + x^2 - 3xe^x, \ \frac{dy}{dt} = -2x + y + 3ye^y$. **5**

8. a) Find all the critical points in the following non-linear system

$\frac{dx}{dt} = 1 - xy, \ \frac{dy}{dt} = x - y^3$.

Determine the nature and stability of each of the critical point. **7**

b) Using Liapunov method determine the stability of the critical point (0, 0) in the following :

i) $\frac{dx}{dt} = y - 2x^3, \frac{dy}{dt} = -2x - 3y^5$

ii) $\frac{dx}{dt} = -x^3 + 3xy^2, \frac{dy}{dt} = x^2y - 4y^3$

iii) $\frac{dx}{dt} = 2xy + x^3, \ \frac{dy}{dt} = 2x^2 - y^3$. **(3+3+3)**