I Semester M.Sc. Degree Examination, January 2017
(R.N.S.) (2011 Onwards)
MATHEMATICS
M – 105 : Discrete Mathematics

Time : 3 Hours Max. Marks : 80

Instructions : *i) Answer* ***any five full*** *questions, choosing atleast* ***two*** *from* ***each*** *Part.*

ii) ***All*** *questions carry* ***equal*** *marks.*

PART – A

1. a) Explain the following :

   i) Rules of inference

   ii) Rule of syllogism

   iii) Modus tollens

   iv) Modus pones

   v) Rule of disjunctive syllogism. **5**

   b) Prove that the following are valid arguments **4**

   i) $p \to (q \to r)$
   $\neg q \to \neg p$
   p
   $\therefore r$

   ii) $\neg p \leftrightarrow q$
   $q \to r$
   $\neg r$
   $\therefore p$

   c) Write down the following proposition in symbolic form and find its negation

   "If all triangles are right angled, then no triangle is equiangular". **3**

   d) Consider the following open statements with the set of all real numbers as the universe

   $p(x) : x \geq 0$; $q(x) : x^2 \geq 0$; $r(x) : x^2 - 3x - 4 = 0$ and $s(x) : x^2 - 3 > 0$.

   Determine the truth values of the following statements

   i) $\exists x,\ p(x) \wedge q(x)$

   ii) $\forall x,\ p(x) \to q(x)$

   iii) $\forall x,\ q(x) \to s(x)$

   iv) $\forall x,\ r(x) \to p(x)$ **4**

2. a) Find the number of integer solutions of $x_1 + x_2 + x_3 + x_4 + x_5 = 30$, where $x_1 \geq 2, x_2 \geq 3, x_3 \geq 4, x_4 \geq 2, x_5 \geq 0$. **3**

b) Write short note on Catalan numbers and Ramsay numbers. **5**

c) How many integers between 1 and 300 are

i) divisible by atleast one of 5, 6, 8

ii) divisible by none of 5, 6, 8. **5**

d) Prove that every set of 37 positive numbers contains atleast two integers that have the same remainder upon divisible by 36. **3**

3. a) Find the recurrence relation and the initial condition for the sequence 0, 2, 6, 12, 20, 30, 42, ... Hence find the general term of the sequence. **4**

b) Tower of Hanoi problem : r circular rings of tapering sizes are slipped on to a peg with the largest ring at the bottom. These rings are to be transferred one at a time onto a another peg and there is a third peg available on which rings can be left temporarily. If, during process of transferring the rings, no ring may be placed on top of a smaller one, in how many moves can these rings be transferred with their relative positions unchanged ? **6**

c) Using generating function, find the number of

i) non-negative and

ii) positive integer solutions of the equation $x_1 + x_2 + x_3 + x_4 = 25$. **6**

4. a) Define :

a) Equivalence relation

b) Zero-one matrices.

Let s be the set of all non-zero integers and A = s×s. On, A define the relation R by (a, b) R (c, d) if and only if ad = bc. Show that R is an equivalence relation. **5**

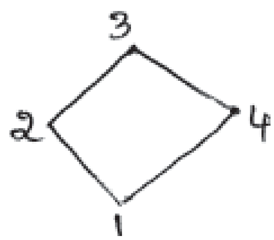
b) Let A = {1, 2, 3, 4, 6, 12}. On A, define the relation R by a R b if and only if a divides b. Prove that R is a partial order on A. Draw the Hasse diagram for this relation. **6**

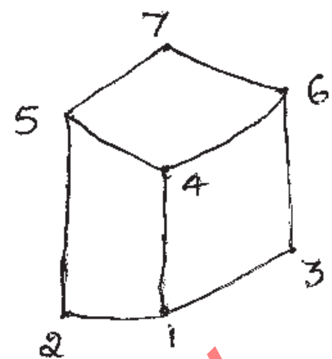
c) Define the terms : **5**

i) Lattice

ii) Bounded lattice and

iii) Distributive lattice. Which of the following Hasse diagrams represent lattice ?



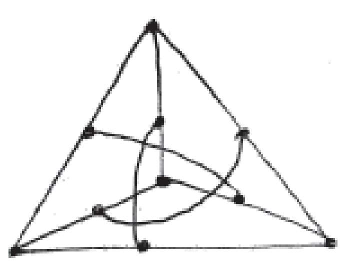


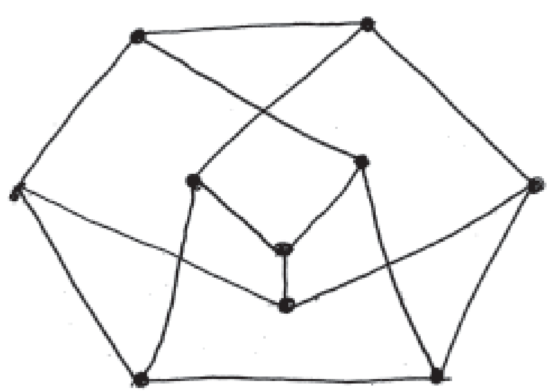
PART – B

5\. a) Define a connected graph. If $q > \frac{1}{2}(p-1)(p-2)$ then prove that a simple (p, q) graph is connected. **5**

b) If G is a simple graph in which every vertex has degree atleast K, then prove that G contains a path of length at least K. Hence, deduce that for $K \geq 2$, G contains a cycle of length atleast K + 1. **6**

c) What are isomorphic graphs ? Determine whether the following graphs are isomorphic. **5**

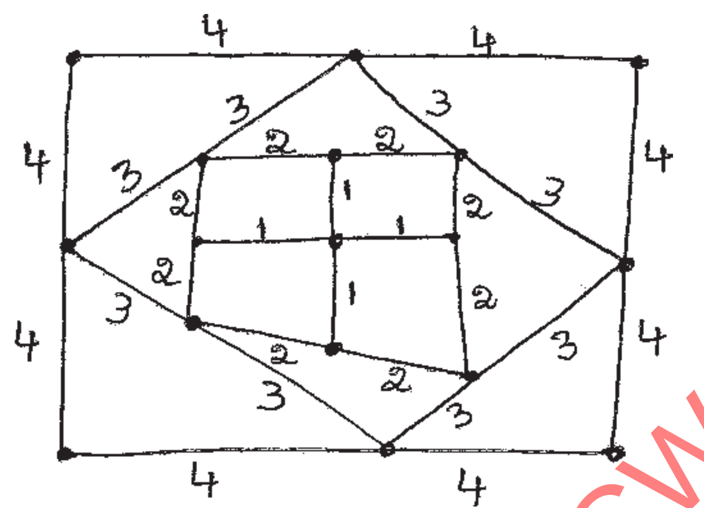




6. a) Define centre of a graph. Prove that the centre of a tree is either K_1 or K_2. **5**

b) Define a spanning tree. Prove that a graph is connected if and only if it has a spanning tree. **5**

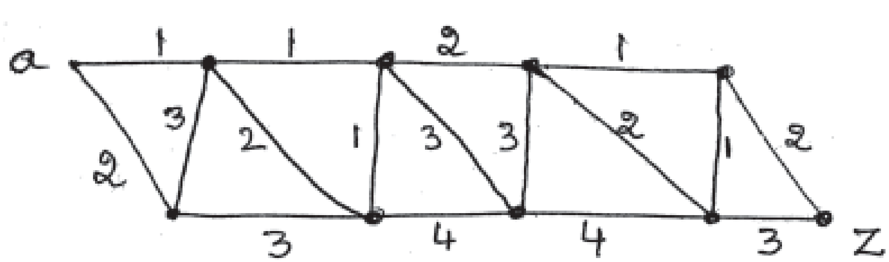
c) Find the minimum spanning tree of the following graph using Prim's algorithm. **6**



7. a) Define a bipartite graph. Find under what condition the complete bipartite graph $K_{m,n}$ has an Eulerian cycle, explain. **5**

b) Prove that a simple graph with p-vertices, p >2 is Hamiltonian if the degree of every vertex is greater than or equal to p/2. **5**

c) Find a shortest distance path between the vertices 'a' and 'z' using Dijkstra's algorithm. **6**



8. a) Define a binary tree. Describe binary search tree algorithm with an example. **6**

b) Define planar graph. Prove that Kuratowski's two graphs are non-planar. **5**

c) State and prove Euler's polyhedron formula. **5**