PG – 966

First Semester M.Sc. Degree Examination, January 2015
(Y2K – 11 (RNS) Scheme)
MATHEMATICS
M – 102 : Real Analysis

Time : 3 Hours   Max. Marks : 80

***Instructions**: 1) Answer **any five** questions choosing atleast **one** from **each** Part.*
*2) **All** questions carry **equal** marks.*

PART – A

1. a) Show that $f(x) = -x^2$ belongs to $R[o, c]$. **4**

   b) Prove that $f \in R[\alpha]$ on $[a, b]$ if and only if given $\in > 0$, there exists a partition P of $[a, b]$ such that : $U(P, f, \alpha) - L(P, f, \alpha) < \in$. **6**

   c) If $f \in R[\alpha]$ on $[a, b]$ and $C \in \mathbb{R}^+$, then prove that $Cf \in R[\alpha]$ on $[a, b]$. **6**

2. a) If f is continuous on [a,b] and α is monotonically increasing function on [a,b], show that $f \in R[\alpha]$. **6**

   b) If f (x) is continuous on [a, b] α (x) be monotonic on [a, b] prove that

   $\int_a^b f\,d\alpha = f(b)\,\alpha(b) - f(a)\,\alpha(a) - \alpha(\xi)\,[f(b) - f(a)]$ where $\xi \in (a,b)$. **6**

   c) If $f_1, f_2 \in R[\alpha]$ on [a,b] and $f_1 \leq f_2$ then show that $\int_a^b f_1\,d\alpha \leq \int_a^b f_2\,d\alpha$. **4**

3. a) If $f \in R[a, b]$ and if $F(x) = \int_a^x f(t)\,dt$ where $a \leq x \leq b$ then prove that F is continuous on [a, b]. Further prove that if f (t) is continuous at a point x_0 in [a,b], then F is differentiable at x_0 and $F^1(x_0) = f(x_0)$. **8**

   b) If $\underset{\mu(p)\to 0}{\text{lt}} S(P, f, \alpha)$ exists then prove that $f \in R[\alpha]$ on [a, b] and that $\underset{\mu(p)\to 0}{\text{lt}}$

   $S(P, f, \alpha) = \int_a^b f\,dx$. **3**

   c) Define a function of bounded variation. Prove that a function of bounded variation on [a, b] is bounded. **5**

P.T.O.

PART – B

4. a) Lef $\{f_n(x)\}$ be a sequence of functions converges to f(x) defined on [a, b] and $M_n = \sup_{x \in [a,b]} |f_n(x) - f(x)|$ then prove that $\{f_n(x)\}$ converges to f (x) uniformly on [a, b] if and only if $M_n \to 0$ as $n \to \infty$. **6**

b) Show that $\{\tan^{-1}(nx)\}$ is uniformly convergent on [a, b], a # 0 but not uniformly convergent on [0, b]. **4**

c) Suppose $f_n \to f$ uniformly on [a, b] and if $x_0 \in$ [a, b] such that $\lim_{x \to x_0} f_n(x) = A_n$, n =1, 2, 3,... Then prove that

i) A_n converges

ii) $\underset{x \to x_0}{\text{lt}} \ \underset{n \to \infty}{\text{lt}} f_n(x) = \underset{n \to \infty}{\text{lt}} \ \underset{x \to x_0}{\text{lt}} f_n(x)$. **6**

5. a) If $\sum_{n=1}^{\infty} f_n(x)$ is uniformly convergent to f(x) on [a, b] and each $f_n(x) \in R[\alpha]$ on [a,b], then prove that $f(x) \in R[\alpha]$ on [a, b]. **8**

b) Let $\{f_n(x)\}$ be a sequence of differentiable functions such that the sequence converges for atleast one point $t \in$ [a,b]. If the sequence of the derivatives of $f_n(x)$, that is $\{f_n^1(x)\}$ is uniformly convergent to F (x) on [a, b], then prove that $[f_n(x)\}$ is uniformly convergent to f (x) on [a, b] and that $f^1(x) = F(x) \ \forall \ x \in$ [a,b]. **8**

6. a) If A is a subset of IR^k, then prove that the following statement are equivalent :

i) A is closed and bounded

ii) A is compact

iii) A is countably compact. **8**

b) State and prove Stone Weierstrass theorem. **8**

PART – C

7. a) Let E be an open subset of $\mathbb{R}^n$ and $f : E \to \mathbb{R}^m$ be a differentiable function at $x_0 \in E$. Then prove that f is continuous at x_0 and $f^1(x_0)$ is unique. **6**

b) If $T_1, T_2 \in L(\mathbb{R}^n, \mathbb{R}^m)$, then prove that

i) $\|T_1 + T_2\| \leq \|T_1\| + \|T_2\|$.

ii) $\|\alpha T_1\| = |\alpha| \; \|T_1\|$. **4**

c) If $\phi : X \to X$ is a contraction on a complete metric space X, then prove that ϕ has a unique fixed point. **6**

8. State and prove the implicit function theorem. **16**