PG – 969

I Semester M.Sc. Degree Examination, January 2015
(Y2K11 (RNS) Scheme)
MATHEMATICS
M105 : Discrete Mathematics

Time : 3 Hours Max. Marks : 80

***Instructions :** Answer **any 5 full** questions, choosing atleast **2** from each Part.*

PART – A

1. a) Explain the methods proof and disproof. **4**

   b) Prove the following statement by contradiction, $\sqrt{5}$ is not a rational number. **4**

   c) Test the validity of the following arguments "If I drive to work then I will arrive in time. I do not drive to work. Therefore, I will not arrive in time." **4**

   d) A computer company requires 30 programmers to handle systems programming jobs and 40 programmers for application programming. If the company appoints 55 programmers to carry out these jobs, how many of these perform jobs of both types ? How many handle only system programming jobs ? **4**

2. a) Suppose that a patient is given a prescription of 45 pills with instruction to take atleast one pill a day for 30 days. Prove that there must be a period of consecutive days during which the patient takes a total of exactly 14 pills. **5**

   b) Find the recurrence relation for the number of binary sequences of length n that have no consecutive 0's. **5**

   c) In how many ways can eight identical cookies be distributed among 3 children if each child receives atleast two cookies but not more than four ? **6**

3. a) Find the co-efficient of x^{18} in $(x + x^2 + x^3 + x^4 + x^5)\,(x^2 + x^3 + x^4 + \ldots)^5$. **5**

   b) A ship carries 48 flags, 12 each of the colors red, white, blue and black. 12 of these flags are placed on a vertical pole in order to communicate a signal to other ships. How many of these signals use an even number of blue flags and odd number of black flags ? **5**

   c) Solve the following recurrence relation using generating functions : $a_{n+2} - 5a_{n+1} + 6a_n = 2$ given $a_0 = 3$, $a_1 = 7$. **6**

P.T.O.

4. a) Define connectivity relation. If R is a relation on a finite set A with $|A| = n$, then prove that $R^{\infty} = R \cup R^2 \cup R^3 \cup \ldots \cup R^n$. **4**

b) Give the step-by-step procedure of Warshall's algorithm. Using this algorithm find the transitive closure of the relation whose matrix is given as

$$M_R = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}$$ defined on the set $A = \{a, b, c, d\}$ **4**

c) Let A be a finite non-empty poset with partial order $\underline{\alpha}$. Then prove that A has atleast one maximal element and at least one minimal element. **4**

d) Define a boolean algebra. In a boolean algebra, for any 2 elements 'a' and 'b' prove that a = b iff $(a \wedge b') \vee (a' \wedge b) = 0$. **4**

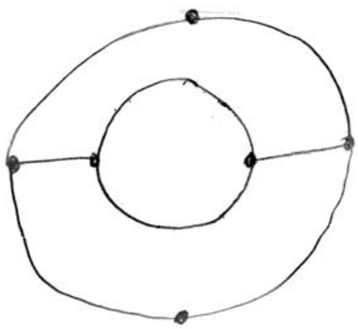
PART – B

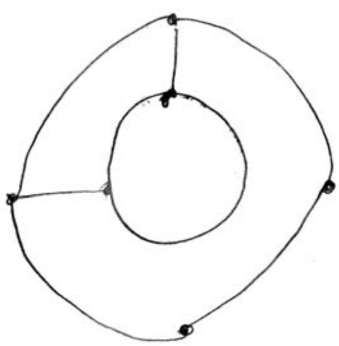
5. a) Define a graph. Prove that a simple graph of minimum degree two has a cycle. **4**

b) Define complement of a graph. Prove that for any graph G with 6 vertices G or $\overline{G}$ contains a triangle. **4**

c) Define a cut-vertex and a cut-edge. Prove that a vertex v is a cut-vertex if there exists two distinct vertices x and y such that v lies on every x – y path. **4**

d) Define graph isomorphism. Are the following graphs isomorphic ? Justify your answer. **4**

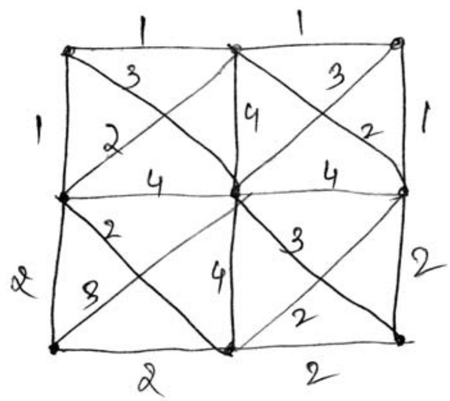




6. a) Define a component in a graph. Prove that a simple graph with p vertices and k components can have at most $\frac{(p-k)(p-k+1)}{2}$ edges. **5**

b) Define radius, diameter, center and periphery of a graph. Prove that rad (G) $\leq$ diam (G) $\leq$ 2 rad (G). **5**

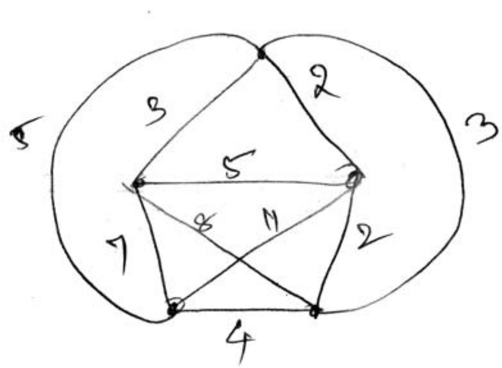
c) Apply prim's algorithm to determine a minimal spanning tree for the weighted graph shown below. **6**



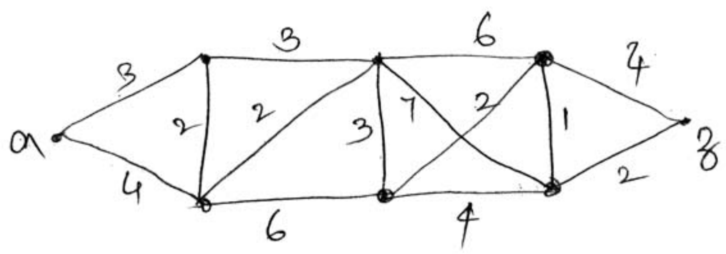
7. a) Define an Eulerian graph. Prove that a connected graph G is an Eulerian graph if and only if G can be decomposed into edge-disjoint cycles. **5**

b) Define a Hamiltonian graph. Show that any k-regular simple graph with 2k – 1 vertices is Hamiltonian. **6**

c) Find a minimum weighted spanning cycle for the following graphs using nearest neighbour method. **5**



8. a) Find a shortest distance path from ‘a’ to ‘z’ using Dijkstra’s algorithm for the following graph. **6**



b) Define a planar graph. If G is a planar connected (p, q) – graph, without triangles, then prove that $q \leq 2p - 4$, whenever $p \geq 3$. **5**

c) Define vertex and edge connectivity of a graph. Prove the following identity with normal notations. $K(G) \leq \lambda(G) \leq \delta(G)$. **5**