PG – 610

I Semester M.Sc. Degree Examination, January 2015
(CBCS)
MATHEMATICS
M105T : Discrete Mathematics

Time : 3 Hours Max. Marks : 70

Instructions : *1) Answer **any five full** questions.*
*2) **All** questions carry **equal** marks.*

1. a) Explain the contradiction method of proof. Use it to prove that $\sqrt{2}$ is irrational. **4**

   b) Test the validity of the following argument.
   "If two sides of a triangle are equal, then the opposite angles are equal". "Two sides of a triangle are not equal". "Therefore, the opposite angles are not equal". **5**

   c) Prove that if 101 integers are selected from the set S = {1, 2, 3, ..., 200}, then at least two of these are such that one divides the other. **5**

2. a) How many five-card hands can be formed from a standard 52-card deck ? If a 5-card hand is chosen at random, what is the probability of obtaining a flush ? What is the probability of obtaining 3, but not 4 aces ? (A hand is a subset and a flush is a hand of all cards of the same suit). **4**

   b) In how many ways can we distribute 24 pencils to 4 children so that each child gets at least 3 pencils, but not more than eight ? **5**

   c) A company appoints 11 software engineers, each of whom to be assigned to one of four offices of the company. Each office should get one of these engineers. In how many ways can these assignments be made ? **5**

3. a) Solve the recurrence relation $a_{n+2}^2 - 5a_{n+1}^2 + 4a_n^2 = 0$ for $n \geq 0$, with initial conditions $a_0 = 4$ and $a_1 = 13$. **4**

   b) Model the 'rabit population' problem as a recurrence relation and solve it explicitly. **5**

   c) Solve the following recurrence relation using generating functions.
   $a_{n+2} - 5a_{n+1} + 6a_n = 2$ given $a_0 = 3$ and $a_1 = 7$. **5**

P.T.O.

4. a) Let A = { lines in a plane}. Let a relation R be defined on A such that xRy if and only if x is parallel to y. Determine the nature of R. **4**

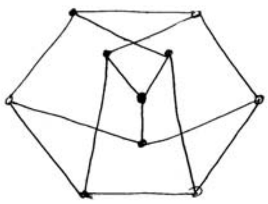
b) Let R be a relation whose matrix representation is as follows. Find the transitive closure of R by using Warshall's algorithm. **5**

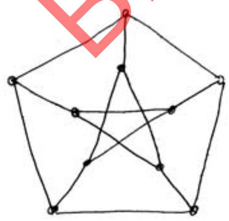
$$M_R = \begin{bmatrix} 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

c) Define a Boolean algebra. Show that in a Boolean algebra, for any two elements 'x' and 'y', x = y if and only if $(x \wedge y') \vee (x' \wedge y) = 0$. Also show that $x' \vee y = 1$ and $x \wedge y' = 0$. **5**

5. a) Define a graph. Prove that in every graph the number of odd degree vertices in even. **4**

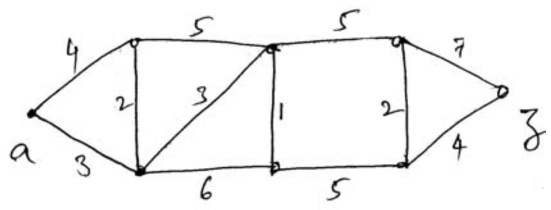
b) Define graph isomorphism. Verify that the following graphs are isomorphic. **5**





c) Define a component in a graph. Prove that a simple graph with p vertices and k components can have at most $\frac{(p-k)(p-k+1)}{2}$ edges. **5**

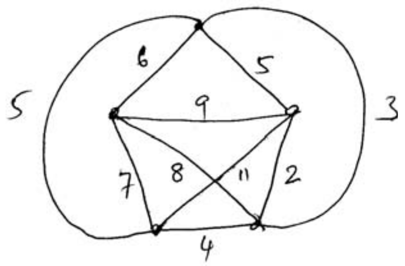
6. a) Find a shortest path from 'a' to 'z' using Dijkstra's algorithm for the following weighted graph. **5**



b) Define an Eulerian graph. Find under which conditions the complete bipartite graph, $K_{m,n}$, has an Eulerian cycle. Explain. **4**

c) Define a Hamiltonian graph. Prove that in a simple graph with p vertices, p>2, is Hamiltonian if the degree of every vertex is greater than or equal to $p/2$. **5**

7. a) Using nearest neighbor method, find a spanning cycle for the following graph. **4**



b) Define a planar graph. If G is planar connected graph with order p, size q, without triangles, then prove that $q \leq 2p - 4$, whenever $p \geq 3$. **4**

c) Define vertex covering, edge covering number of a graph. Show that

$\alpha_0(G) + \beta_0(G) = p = \alpha_1(G) + \beta_1(G)$. **6**

8. a) Show that a (p – q)-graph is a tree if and only if it is acyclic and q = p – 1. **4**

b) Define center, periphery, radius and diameter of a graph G. Prove that the center of a tree is either K_1 or K_2. **5**

c) Write and apply Kruskal's algorithm to determine a minimum spanning tree for the weighted graph as shown below. **5**