PG – 965

First Semester M.Sc. Examination, January 2015
(Y2K11 (RNS) Scheme)
MATHEMATICS
M 101 : Algebra – I

Time : 3 Hours Max. Marks : 80

Instructions : *1) Answer **any 5** questions, choosing atleast **2** from **each** Part.*
*2) **All** questions carry **equal** marks.*

PART – A

1. a) Define permutation on a set. Show that every permutation on a finite set is a product of disjoint cycles. **5**

   b) Let $\phi : G \to \overline{G}$ be an epimorphism with kernel K and let N be a normal subgroup of G. Then prove that $\frac{G/K}{N/K} \approx G/N$. **5**

   c) Show that every group is isomorphic to a subgroup of A (S), for some appropriate S. **6**

2. a) State and prove the orbit-stabilizer theorem. **5**

   b) Derive the class equation for finite groups. **6**

   c) By using the generator-relator form of D_8 ; find the conjugacy class of all the elements of the dehydral group D_8. **5**

3. a) State and prove the Cauchy's theorem for abelian groups. **6**

   b) Show that any two p-sylow subgroups are conjugate to each other. **6**

   c) Let o (G) = pq, where p and q are distinct primes with p < q and $q \not\equiv 1 \pmod p$. Then prove that G is cyclic. **4**

4. a) Define a solvable group. Prove that every subgroup of a solvable group is solvable. **6**

   b) Show that a normal subgroup N of G is maximal if and only if the quotient group G/N is simple. **6**

   c) Show that the symmetric group S_3 is not simple. **4**

P.T.O.

PART – B

5. a) Define integral domain and field. Prove that every finite integral domain is a field. **6**

b) If U is an ideal of a ring R, let $[R : U] = \{x \in R : r\,x \in U \ \forall\ r \in R\}$. Prove that $[R : U]$ is an ideal of R containing U. **5**

c) Show that a homomorphism ϕ of a ring R onto a ring R' is an isomorphism if and only if $\ker\phi = \{0\}$. **5**

6. a) Show that a ring $\mathbb{Z}$ of integers is a principle ideal ring. **4**

b) Define maximal ideal of a ring. If R is a commutative ring with unit element and M is an ideal of R, then show that M is a maximal ideal of R if and only if R/M is a field. **6**

c) Prove that in a principal ideal ring, every non-zero prime ideal is maximal ideal. **6**

7. a) Define field of quotients of an integral domain D. Show that any two isomorphic integral domains have isomorphic quotient fields. **6**

b) Let R be a euclidean ring. Then show that any two elements a and b in R have a gcd 'd'. More over $d = \lambda a + \mu b$ for some $\lambda, \ \mu \in R$. **4**

c) If p is a prime number of the form 4n+1, prove that $p = a^2 + b^2$ for some integers a, b. **6**

8. a) Define an irreducible polynomial in F [X]. Explain by an example that an ideal generated by an irreducible polynomial in F [X] is a maximal ideal in F [X]. **6**

b) State and prove Einstein criterion for irreducibility of a polynomial. **6**

c) If p is a prime number, prove that the polynomial $x^n - p$ is irreducible over Q. **4**