PG – 378

I Semester M.Sc. Degree Examination, Jan./Feb. 2014
(NS Scheme)
MATHEMATICS
M-104 : Ordinary Differential Equations

Time : 3 Hours　　　　Max. Marks : 80

Instructions : *i) Answer **any five** questions choosing at least **two** from **each** Part.*
*ii) **All** questions carry **equal** marks.*

1. a) If $\{\phi_j(x), j = 1 \text{ to } n\}$ is a fundamental set of $L_n y = 0$ then show that $\{\psi_j(x), j = 1 \text{ to } n\}$ also form a fundamental set of $L_n y = 0$ iff there exists a non-singular matrix C such that $[\psi_1, \psi_2, \ldots \psi_n]^T = C\,[\phi_1, \phi_2, \ldots \phi_n]^T$.

   Also, deduce that

   $$\frac{W\{\psi_j(x), j = 1 \text{ to } n\}}{W\{\phi_j(x), j = 1 \text{ to } n\}} = \operatorname{Det}(C).$$ 12

   b) Find the Wronskian of the independent solutions of
   $Y^{(4)} - 4y^{(3)} + 6y^{(2)} - 4y' + y = 0.$ 4

2. a) Prove that

   i) $L_n^{**} = L_n$

   ii) $L = \frac{d^k}{dx^k} p(x) \frac{d^k}{dx^k}$ is a self-adjoint operator, where p(x) is a real valued function. 10

   b) Find the general solution of $x^2y'' + 7xy' + 8y = 0$
   by finding the solution of its adjoint equation. 6

3. a) State and prove Sturm's comparison theorem on the zeros of the solutions of self-adjoint differential equations. Illustrate the result with an example. 8

   b) Show that the equation

   $$y'' + k\left\{\frac{1}{4x^2} + \frac{1}{(x \log x)^2}\right\}y = 0$$

   is oscillatory or non-oscillatory as $k > \frac{1}{4}$ or $k \leq \frac{1}{4}$. 8

P.T.O.

4. a) Define Sturm-Liouville problem. 3

b) Find the eigen values and eigen functions of

$(xy')' + \frac{\lambda}{x} y = 0$; $y'(1) = 0 = y'(e^{2\pi})$, where λ is non-negative. 7

c) Obtain the Green's function for $y'' + \lambda y = x^2$; $y(0) = 0 = y(\pi)$. 6

PART – B

5. a) Define :

i) Ordinary point

ii) Singular point

iii) Regular singular point

of $y'' + P(x)y' + Q(x)y = 0$. Obtain the series solution of the equation

$$y'' + x^2 y' + 2xy = 0$$

about x = 0. 5

b) Find the general power series solution of the equation

$$x^2 y'' - 2xy' + (2 - x^2)y = 0$$

about x = 0. 6

c) Prove that : $\frac{1}{1-t} e^{\frac{-xt}{1-t}} = \sum_{n=0}^{\infty} L_n(x) t^n$. 5

6. a) Apply Frobenius method to find the general solution of the equation $x(1 - x)y'' + [c - (a + b + 1)x]y' - aby = 0$ about the regular singular point x = 0. 7

b) Find the general power series solution of $(x^2 - 1)y'' + (5x + 4)y' + 4y = 0$ about x = 1. 5

c) Prove that :

$$\int_{-1}^{1} \frac{T_m(x) T_n(x)}{\sqrt{1-x^2}} dx = \begin{cases} 0, & \text{if } m \neq n \\ \frac{\pi}{2}, & \text{if } m = n \neq 0 \\ \pi, & \text{if } m = n = 0 \end{cases}$$ 4

7. a) Find the fundamental matrix solution of

$$\frac{dx}{dt} = \begin{bmatrix} 0 & -2 & 6 \\ -2 & 9 & -12 \\ -1 & 2 & -1 \end{bmatrix} X.$$ 5

b) Explain the four types of critical points of

$$\frac{dx}{dt} = ax + by$$

$$\frac{dy}{dt} = cx + dy,$$

$ad - bc \neq 0.$ 5

c) Determine the nature and stability of critical point of

i) $\frac{dx}{dt} = x - 4y\,;\ \frac{dy}{dt} = 2x - 5y$

ii) $\frac{dx}{dt} = ax + y,\ \frac{dy}{dt} = x - ay\,(a \neq 0).$ 6

8. a) Find all the critical points of the nonlinear system.

$$\frac{dx}{dt} = 1 - xy\,;\ \frac{dy}{dt} = x - y^3.$$

Determine the nature and stability of each of them. 6

b) Determine the nature and stability of the critical point (0, 0) of the nonlinear equation

$$\frac{d^2x}{dt^2} + \mu(x^2 - 1)\frac{dx}{dt} + x = 0$$

When $\mu > 0$ and $\mu < 0$. 5

c) Using the Liapunov method, determine the stability of the critical point (0, 0) of

$$\frac{dx}{dt} = -x^3 - 3xy^4$$

$$\frac{dy}{dt} = x^2y - 2y^3 - y^5.$$ 5
