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I Semester B.A./B.Sc. Examination, December - 2019
(CBCS) (Semester Scheme) (F+R) (2014-15 and Onwards)
MATHEMATICS - I

Time : 3 Hours   Max. Marks : 70

Instruction : *Answer **all** questions.*

PART - A

Answer **any five** sub-questions.   **5x2=10**

1. (a) If λ is an eigen value of a non-singular matrix A, then show that λ^{-1} is an eigen value of A^{-1}.

(b) Find the eigen values of the matrix $A=\begin{pmatrix}-3 & 8\\ -2 & 7\end{pmatrix}$

(c) Find the n^{th} derivative of $\sin^2 x$.

(d) If $z = x^2+y^2-3xy$, then prove that $\frac{\partial^2 z}{\partial x^2} = \frac{\partial^2 z}{\partial y^2}$

(e) Evaluate : $\int_0^{\frac{\pi}{2}} \cos^3 x \, dx$

(f) Evaluate : $\int_0^{\frac{\pi}{2}} \sin^7 x \cos^4 x \, dx$

(g) Find k so that the spheres $x^2+y^2+z^2+6y+2z+k = 0$ and $x^2+y^2+z^2+6x+8y+4z+20=0$ cuts orthogonally.

(h) Show that the plane $x+2y-3z+4=0$ is perpendicular to each of the planes $2x+5y+4z+1=0$ and $4x+7y+6z+2=0$

PART - B

Answer **one full** question.   **1x15=15**

2. (a) Find the rank of the matrix $A=\begin{pmatrix}1 & 1 & 1 & 6\\ 1 & -1 & 2 & 5\\ 3 & 1 & 1 & 8\\ 2 & -2 & 3 & 7\end{pmatrix}$ by reducing it to echelon form.

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(b) Show that the system of equations $x+y+2z=a$, $x+3y-2z=b$, $5x+7y+6z=c$ is consistent only when $c=4a+b$. Assuming this condition express x, y in terms of a, b, z.

(c) Using Cayley-Hamilton theorem, find the inverse of the matrix

$$A=\begin{pmatrix}1 & 0 & 2\\ 0 & 2 & 1\\ 2 & 0 & 3\end{pmatrix}$$

OR

3. (a) Find the rank of the matrix $A=\begin{pmatrix}1 & 1 & -1\\ 2 & -3 & 4\\ 3 & -2 & 3\end{pmatrix}$ by reducing it to normal form.

(b) Solve completely the system of equations :

$$x_1+3x_2+2x_3=0$$
$$2x_1-x_2+3x_3=0$$
$$3x_1-5x_2+4x_3=0$$
$$x_1+17x_2+4x_3=0$$

(c) Find eigen values and eigen vectors of the matrix $A=\begin{pmatrix}5 & -1\\ 4 & 9\end{pmatrix}$

PART - C

Answer **two full** questions. **2x15=30**

4. (a) Find the n^{th} derivative of $\dfrac{x+3}{(x-1)(x+2)}$

(b) Find the n^{th} derivative of $\sin^2 x\cos^3 x$

(c) If $y=e^{m\sin^{-1}x}$, then show that $(1-x^2)y_{n+2}-(2n+1)xy_{n+1}-(n^2+m^2)y_n=0$

OR

5. (a) If $u=\log(x^3+y^3+z^3-3xyz)$, then show that $\dfrac{\partial u}{\partial x}+\dfrac{\partial u}{\partial y}+\dfrac{\partial u}{\partial z}=\dfrac{3}{x+y+z}$

(b) State and prove Euler's theorem for homogeneous function.

(c) Find $\dfrac{\partial u}{\partial t}$, if $u=xy^2+x^2y$, where $x=at^2$ and $y=2at$.

6. (a) If $u = \sin^{-1}\left(\frac{x^2+y^2}{x+y}\right)$, then show that $x\frac{\partial u}{\partial x} = y\frac{\partial u}{\partial y} = \tan u$

(b) Verify Euler's theorem for $u = ax^2 + 2hxy + by^2$

(c) Obtain reduction for $\int \cot^n x\,dx$ and hence evaluate $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot^6 x\,dx$]

OR

7. (a) Obtain the reduction formula for $\int \sec^n x\,dx$

(b) Evaluate $\int_0^1 \frac{x^6}{\sqrt{1-x^2}}\,dx$

(c) Evaluate $\int_0^1 \frac{x^a-1}{\log x}\,dx$, where a is a parameter, using differentiation under integral sign.

PART - D

Answer **one full** question. **1x15=15**

8. (a) Find the equation of the plane passing through the line of intersection of the planes $x+2y+3z=4$, $2x+y-z+5=0$ and perpendicular to the plane $5x+3y+6z+8=0$.

(b) Show that the lines $\frac{x-1}{2} = \frac{y-2}{2} = \frac{z-3}{1}$ and $\frac{x-2}{3} = \frac{y-2}{2} = \frac{z-6}{4}$ are coplanar. Find the equation of the plane containing these lines.

(c) Find the equation of the sphere passing through the points (3, 0, 0), (0, – 1, 0), (0, 0, –2) and having its centre on the plane $3x+2y+4z-1=0$

OR

9. (a) Find the length and equation of the shortest distance between the lines $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$ and $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$.

(b) Find the equation of the right circular cone which passes through the point (1, 1, 2) and has its vertex at the origin and axis is the line $\frac{x}{2} = -\frac{y}{4} = \frac{z}{3}$

(c) Find the equation of the right circular cylinder generated by revolving the line $\frac{x-1}{2} = \frac{y-3}{2} = \frac{z-5}{-1}$ about the line $\frac{x+1}{2} = \frac{y+3}{2} = \frac{z+5}{-1}$

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