11123

560

I Semester B.Sc. Examination, December 2018
(CBCS Scheme)
MATHEMATICS – I

Time : 3 Hours Max. Marks : 70

Instruction : *Answer **all** questions.*

PART – A

Answer **any five** questions : **(5×2=10)**

1. a) Find the value of 'a' in order that the matrix $A = \begin{bmatrix} 6 & a & -1 \\ 2 & 3 & 1 \\ 3 & 4 & 2 \end{bmatrix}$ is of rank 2.

   b) Find the eigen values of the Matrix $A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$.

   c) Find the n^{th} derivative of $\frac{1}{(5x-2)^3}$.

   d) If $f(x, y) = \tan^{-1}\left(\frac{x}{y}\right)$, find $\frac{\partial f}{\partial x}$.

   e) Evaluate $\int_0^{\pi/4} \tan^5 x \, dx$.

   f) Evaluate $\int_0^{\pi/2} \sin^5\theta \cos^3\theta \, d\theta$.

   g) Find the angle between the line $\frac{x-3}{2} = \frac{y+1}{-1} = \frac{z+4}{3}$ and the plane $2x + 3y - z - 4 = 0$.

   h) Find the centre and radius of the sphere $x^2 + y^2 + z^2 - 4x + 4y + 5 = 0$.

P.T.O.

PART – B

Answer **one full** question : (1×15=15)

2. a) Find the rank of the matrix $\begin{bmatrix} 1 & 3 & -1 & 2 \\ 0 & 11 & -5 & 3 \\ 2 & -5 & 3 & 1 \\ 4 & 1 & 1 & 5 \end{bmatrix}$ by reducing it to echelon form.

b) Find the real value of λ for which the system $x + 2y + 3z = \lambda x$, $3x + y + 2z = \lambda y$, $2x + 3y + z = \lambda z$ have non zero solutions.

c) State and prove Cayley-Hamilton theorem.

OR

3. a) Reduce the matrix $\begin{bmatrix} 1 & 4 & 1 & 2 \\ 2 & 1 & -3 & 6 \\ 3 & -3 & 1 & 2 \end{bmatrix}$ to its normal form and find its rank.

b) Test the system of equations $x + 2y - z = 3$, $3x - y + 2z = 1$, $2x - 2y + 3z = 2$ for consistency and solve if it is consistent.

c) Find the eigen values and the corresponding eigen vector of the matrix $\begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$.

PART – C

Answer **two full** questions : (2×15=30)

4. a) Find the n^{th} derivative of $\frac{x+2}{2x^2+3x+1}$.

b) Find the n^{th} derivatives of

i) $x^2 \cos x$

ii) $\log(2x + 1)$.

c) If $y = e^{m \sin^{-1} x}$ then show that $(1 - x^2)y_{n+2} - (2n + 1)\, xy_{n+1} - (n^2 + m^2)\, y_n = 0$.

OR

5. a) If $u = e^x (x \sin y + y \cos y)$, then prove that $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$.

   b) If $u = \tan^{-1}\left(\frac{x^3 + y^3}{x + y}\right)$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$.

   c) Find $\frac{df}{dt}$ where $f(x, y) = \frac{x}{y} + \frac{y}{x}$, $x = t$, $y = t + 1$ using partial differentiation.

6. a) If $u = f(x - y, y - z, z - x)$, then prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$.

   b) If $x = r\cos\theta$, $y = r\sin\theta$, then prove that $\frac{\partial(x,y)}{\partial(r,\theta)} \times \frac{\partial(r,\theta)}{\partial(x,y)} = 1$.

   c) Obtain reduction formula for $\int \sin^n x\, dx$ where n is a positive integer.

OR

7. a) Obtain reduction formula for $\int \sec^n x\, dx$ where n is a positive integer.

   b) Evaluate $\int_0^1 \frac{x^3}{\sqrt{1-x^2}}\, dx$.

   c) Using Leibnitz's rule of differentiation under integral sign evaluate $\int_0^1 \frac{x^\alpha - 1}{\log x}\, dx$ where $\alpha > 0$ is a parameter.

PART – D

Answer **one full** question : **(1×15=15)**

8. a) Find the equation of the plane through the intersection of the planes $x - 2y + z - 7 = 0$ and $2x + 3y - 4z = 0$ and cutting intercept 4 units on the x-axis.

   b) Find the length of the shortest distance between the lines $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$ and $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$

   c) Find the equation of the sphere which passes through the points (1, 0, 0), (0, 1, 0) and (0, 0, 1) and whose centre lies on the plane $3x - y + z = 2$.

OR

9. a) Show that the lines $\frac{x-1}{1}=\frac{y+1}{-1}=\frac{z-3}{1}$ and $\frac{x-2}{2}=\frac{y-4}{1}=\frac{z-6}{3}$ are coplanar. Find also the equation of the plane containing them.

   b) Derive the equation of right circular cone in its standard form $x^2 + y^2 = z^2 \tan^2 \alpha$.

   c) Find the equation of the right circular cylinder of radius 3 units and axis $\frac{x-1}{2}=\frac{y-3}{2}=\frac{z-5}{-1}$.