I Semester B.A./B.Sc. Examination, November/December 2015
(2014-15 & Onwards)
(Semester Scheme) (CBCS) (F+R)
MATHEMATICS – I

Time : 3 Hours

Max. Marks : 70

Instruction : Answer all questions.

PART – A

1. Answer **any five** questions : **(5×2=10)**

   a) Find the value of K in order that the matrix

   $$A = \begin{bmatrix} 6 & K & -1 \\ 2 & 3 & 1 \\ 3 & 4 & 2 \end{bmatrix} \text{ is of rank 2.}$$

   b) Find the value of λ for which the system of equations
   $2x - y + 2z = 0$, $3x + y - z = 0$ and $\lambda x - 2y + z = 0$ has a non-trivial solution.

   c) Find the nth derivative of $e^{5x} + \sin 5x$.

   d) If $z = x^3 - 3xy^2$ show that $\dfrac{\partial^2 z}{\partial x^2} + \dfrac{\partial^2 z}{\partial y^2} = 0$.

   e) Evaluate $\displaystyle\int_0^{\pi/2} \sin^6 x \, \cos^3 x \, dx$.

   f) Evaluate $\displaystyle\int_0^{\pi/2} \cos^5 \theta \, d\theta$.

   g) Find the angle between the planes $2x - y + 2z - 3 = 0$ and $3x + 6y + 2z - 4 = 0$.

   h) If two spheres $x^2 + y^2 + z^2 + 6z - k = 0$ and $x^2 + y^2 + z^2 + 10y - 4z - 8 = 0$ cut orthogonally. Find k.

PART – B

$(1 \times 15 = 15)$

2. Answer **any one full** question :

a) Find the rank of the matrix

$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \\ 2 & 1 & 3 & 1 \\ 1 & 2 & 3 & -1 \end{bmatrix}$$ by reducing it to echelon form.

b) Solve the system of equations $x + 2y + z = 3$, $2x + 3y + 3z = 10$ and $3x - y + 2z = 13$ by elimination method.

c) State Cayley-Hamilton theorem and find the inverse of the matrix

$$\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$ by using it.

OR

3. a) Find the rank of the matrix

$$\begin{bmatrix} 1 & 1 & 1 & 2 \\ 2 & 1 & -3 & -6 \\ 3 & -3 & 1 & 2 \end{bmatrix}$$ by reducing it to normal form.

b) Find λ and μ such that the system of equations $x + 3y + 4z = 5$, $x + 2y + z = 3$ and $x + 3y + \lambda z = \mu$ has (i) no solution (ii) unique solution (iii) many solutions.

c) Find the eigen values and the corresponding eigen vectors of the matrix

$$\begin{bmatrix} -3 & 8 \\ -2 & 7 \end{bmatrix}.$$

PART – C

$(2 \times 15 = 30)$

4. Answer **any two full** questions :

a) Find the nth derivative of $\dfrac{x}{(x-2)(x+3)}$.

b) Find the nth derivative of

   i) $y = x^2 e^{5x}$ ii) $y = \log(5x + 4)$

c) If $y = \tan^{-1}x$ prove that

   $(1 + x^2)\, y_{n+2} + 2(n + 1)\, xy_{n+1} + n(n+1)y_n = 0$

<div align="center">OR</div>

5. a) If $u = (x - y)^n + (y - z)^n + (z - x)^n$ prove that

   $\dfrac{\partial u}{\partial x} + \dfrac{\partial u}{\partial y} + \dfrac{\partial u}{\partial z} = 0$.

  b) If $u = \tan^{-1}\left(\dfrac{x^3 - y^3}{x + y}\right)$ prove that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = \sin 2u$.

  c) Find $\dfrac{df}{dt}$ where $f(x, y, z) = \log(x^2 + y^2 + z^2)$, $x = e^t$, $y = \sin t$, $z = \cos t$ by using partial differentiation.

6. a) If $z = f(x, y)$, $x = u - v$, $y = uv$, prove that $u\dfrac{\partial z}{\partial u} + v\dfrac{\partial z}{\partial v} = x\dfrac{\partial z}{\partial x} + 2y\dfrac{\partial z}{\partial y}$.

  b) If $u = x + 3y^2 - z^3$, $v = 2x^2 - yz$, $w = 2z^2 - xy$. Evaluate $\dfrac{\partial(u, v, w)}{\partial(x, y, z)}$ at $(1, -1, 0)$.

  c) Obtain the reduction formula for $\int \tan^n x\, dx$.

<div align="center">OR</div>

7. a) Obtain the reduction formula for $\int \cos^n x\, dx$.

  b) Evaluate : $\displaystyle\int_0^1 \dfrac{x^6}{\sqrt{1 - x^2}}\, dx$.

  c) Evaluate : $\displaystyle\int_0^\infty \dfrac{e^{-x}\,\sin\alpha}{x}$ where α is a Parameter using Leibnitz's rule of differentiation under the integral sign.

PART – D

8. Answer **any one full** question. $(1\times15=15)$

  a) Find the equation of the plane passing through the line of intersection of the planes $2x + y + 3z - 4 = 0$ and $4x - y + 2z - 7 = 0$ and perpendicular to the plane $x + 3y - 4z + 6 = 0$.

  b) Prove that the lines $\frac{x-1}{2} = \frac{y-2}{2} = \frac{z-3}{1}$ and $\frac{x-2}{3} = \frac{y-2}{2} = \frac{z-6}{4}$ are coplanar and find the equation of the plane containing them.

  c) Find the equation of the sphere which passes through the points $(1, 0, 0), (0, 1, 0), (0, 0, 1)$ and whose centre lies on the plane $3x - y + z = 2$.

  OR

9. a) Find the shortest distance between the skew lines

   $\frac{x}{2} = \frac{y}{-3} = \frac{z}{1}$ and $\frac{x-2}{3} = \frac{y-1}{-5} = \frac{z+2}{1}$.

  b) Derive the equation of a sign circular cone whose vertex is orgin, axis is z-axis and semi vertical angle is α. Hence obtain the equation of right circular cylinder whose vertex is at origin, axis is z – axis and semi vertical angle is 30°.

  c) Find the equation of right circular cylinder whose radius is 4 units and was passes through $(1, -2, 3)$ and $(3, -1, 1)$.